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Fractal assessment of street-level skylines: a possible means of assessing and comparing character

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Abstract. *Fractal analysis and the calculation of fractal dimension offers the potential for the numerical characterization of places by providing a synthetic measurement of place complexity. This paper provides a fractal analysis of street-scale urban skylines, linking the calculation of fractal dimension to the presence of the physical features making up a skyline. A technique for calculating skyline fractal dimensions is presented and suggestions are made about the use of fractal analysis in comparing the character of places.*

Key Words: skyline, fractal dimension, character, complexity, urban design, urban morphology

There is much current debate in planning and urban design about character and local distinctiveness and the need to design with local identity in mind. Many well-liked urban and village environments are seen as quaint and 'olde worlde', having qualities that Gordon Cullen (1961) described using a 'townscape' approach. These environments often form the template from which designers and conservationists draw their inspiration for creating or enhancing distinctiveness. Such environments comprise mixtures of building style and materials: the building line often has irregular set-back depths, skylines are often broken or undulating and, as they have invariably developed over a long period, the townscapes are weathered and contain mature vegetation. The combination of these features is a place characterized by the irregular, by the higgledy-piggledy, by variation – although usually within a fairly common theme.

It is this variation within a theme that designers are seeking to replicate and to which people seem to respond positively. Perhaps it is this irregularity that contributes to Alexander's (1977) notion of the quality without a name. The difficulty facing designers is how to replicate this combined irregularity: how to deliver a positive degree of variety within a theme without simply producing complications and confusion. One way towards solving this problem is to try to understand the underlying complexity and character of the places we use as inspiration; not slavishly to copy them but to use their underlying complexity to generate new design with a similar level of irregularity (Cooper, 2000).

One method of assessing the irregular is

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through the new sciences of complexity, specifically through the use of fractal geometry and the calculation of fractal dimension. Developed by Mandlebrot (1977), this allows us to measure the irregularity of an object, or, in the case of an urban environment, combination of objects. Once measured, this fractal dimension can be used both to compare numerically the character of existing places and ultimately to generate new patterns with the same underlying degree of complexity.

A number of authors have recently explored the use of fractal geometry in planning, architecture and urban design – Batty (1995), Batty and Longley (1994a, 1994b), Batty and Xie (1994), Bechoefer and Bovill (1995), Bovill (1996), Frankhauser (1994), Jencks (1995), Kavannagh (1992), Mizuno and Kakei (1990), Oku (1990), Peterson (1996), Robertson (1992, 1995) – and a review can be found in Cooper (2000). This type of assessment can be applied to various kinds of 'irregularity': for example, wigglyness in road-layout, façades and building lines. Here we concentrate on one aspect of the urban scene: the skyline.

This exploratory paper shows how fractal dimension can be calculated for a series of street-level urban skylines, and how the resultant numerical measurement can be related to the presence of certain morphological features that, in combination, affect the character of a place. Differences between and along a selection of streets are quantifed using fractal dimension as an illustration of how changes in physical character can be combined and recorded in a single number that allows quick comparison to be made between places.

The fractal measurement of skylines has already been established by Oku (1990) in relation to *distant views* of city-wide skylines. This paper examines skylines viewed *from within the street* at scales relevant to designers with a concern for the visual quality of public space and seeking to create urban places.

The paper is divided into three sections. The first gives a brief description of the concept of fractal dimension. The second describes the method used to calculate the fractal dimensions for a series of case streets in Oxford, UK. The third analyses these fractal dimensions in relation to the physical features comprising the skylines as a means of linking fractal dimension with built character.

Fractal dimension

The concept of fractal dimension allows the degree of irregularity of a shape or object to be measured and represented as a number. This number (D) lies between the Euclidean dimensions of 1, 2, or 3. For example the fractal dimension of an irregular line, such as a coastline, would lie somewhere between 1 and 2. It is not a simple straight line – that would be one-dimensional – but neither is it a full plane which would have two dimensions. It lies somewhere between the two. Fractal dimension can be represented as fractional or non-integer numbers, whereas Euclidean dimensions are integers. Essentially, fractal dimension is a measure of how well a particular object fills the space in which it is drawn. For example, using the concept of theoretical mathematical fractals it is possible to imagine an infinitely long line drawn in a finite space. The line is infinitely folded and irregular on decreasingly small scales. Its length can be infinite as it increases through irregularity: it increases its density within its given space. Figure 1 illustrates the concept in relation to a simple straight line in comparison with two traced skylines and shows how increased 'roughness' of line can be represented numerically (D). Line 'a' is a straight line with a single dimension (length) and is therefore labelled as $D = 1$. Line 'b' has a greater degree of 'roughness' and has a correspondingly higher D value of 1.0245 – it is not a simple straight line. Line 'c' exhibits a further degree of irregularity and has a D value approaching 1.3. These examples show how a numerical value (D) can be used to quantify the degree of irregularity in a line.

Mandelbrot's work on coastline measure-

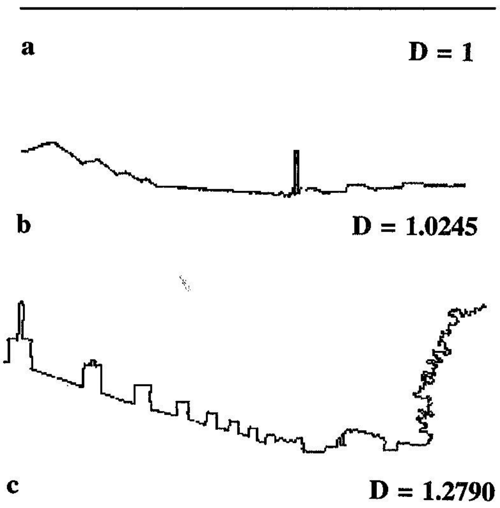

Figure 1. Fractal dimension in relation to line 'roughness'.

ment followed earlier work on national boundaries by Lewis F. Richardson in 1961 (see Mandelbrot, 1977). Richardson experimented in measuring the west coast of Britain and the Spanish-Portuguese land boundary. He had noticed that his results depended on the scale of the maps being used: there were discrepancies of up to 20 per cent in the total lengths. It was this discovery that subsequently led Mandelbrot to develop the concept of fractal dimension. Mandelbrot (1977) argues that the length of a coastline is infinite, with increasing detail picked up as the measurement scale reduces. This is the key to understanding fractal dimension. It is the relationship between measured length and measurement scale that is the basis of calculating fractal dimension.

There are several methods of characterizing the fractal dimension of irregular or rugged lines. Essentially, all of the methods seek to establish a relationship between measured length, surface or volume, and scale, by measuring how length, surface, or volume increases at smaller and smaller scales. The one that will be described and used here is the structured walk or ruler method, where the distance used for each 'step' (the 'detail' of the walk) is related to the scale used.

In its simplest form, the ruler method employs a set of dividers, compasses or rulers set at various predetermined stride lengths (s) to allow measurement at various scales. The rulers are then 'walked' along the subject line or around the perimeter of an object at each of the predetermined settings and the total lengths (p) are recorded.

To compare the results of measurement at different scales, and subsequently to calculate the fractal dimension, the measurements are entered into a double logarithmic graph as the log of s against the log of p, where p is the resultant lengths. This removes the difficulty of reading length-versus-settings relationships when the settings used may vary from several hundred units to just a few. These log/log diagrams are referred to as Richardson plots (after Richardson, 1961).

When points on a log/log diagram fall on a virtual straight line it is safe to assume that a power law relationship exists between the two sets of data (Peitgen *et al.*, 1992, p.192). This allows the exponent of that power law to be read off as the slope of that straight line (d). To arrive at the value of d, the equation $y = dx + b$ can be employed. This is the description of a straight line on an x, y diagram, where b is the intercept point of the straight line on the y axis and d is the gradient of the line. So $d = (y^2 - y^1) / (x^2 - x^1)$ for any pair of points, for example (x^1, y^1) and (x^2, y^2) on a line. To arrive at d, two sets of co-ordinates on the line are selected and subjected to the equation that will give a value for d; the gradient of the line. This value d is essential in calculating the fractal dimension of the subject in question.

Bearing in mind that the fractal dimension is D, this method gives the ruler dimension and so is written Dr. The fractal dimension is then achieved by employing the equation $Dr = 1 + d$. This gives a direct measurement of the roughness of the fractal object by adding d to the topological dimension 1. In effect we know that the subject, in the case of the coastline or skyline, is some kind of line, so its base dimension must be 1. It is also known that it has roughness – it is not a smooth line. So to get the overall effect we add an indication of its roughness to its base dimension – hence we add *d* to its base dimension.

However, for a number of reasons, caution is needed when calculating fractal dimension in this manner. First, it reveals a fractal dimension that is related to observations made over a certain range of scales and relates only to those scales. In terms of urban design, this makes the selection of a useful and useable measurement scale paramount in achieving meaningful results when evaluating different characteristics. Any evaluation of D needs to be undertaken at scales that are meaningful in relation to the particular subject.

Secondly, different regions of a subject may have different fractal properties – commonly referred to as multi-fractality. Batty and Longley (1994a), for example, have observed this multi-fractality in the urban boundary of Cardiff. Koch (1993) illustrates this with the example of the coast of Great Britain, which has some parts that are rougher than others. The east coast is less rugged than the west coast due to differences in the degree of exposure to weathering and differing geology. The two coasts will display differing fractal dimensions: the overall fractal dimension is therefore intermediate between the two, perhaps hiding interesting or significant detail.

Skyline calculation method

In this section, the ruler-measurement is used to assess the characteristics of skylines visible along a series of streets. First, the calculation parameters are explained, then the mean Dr values for the samples are presented, together with an initial explanation for their value in terms of the built and natural form characteristics present in each traced skyline. The initial explanations are subsequently tested in section three using correlation and multiple regression analysis. The levels of variance within the sets of sample traces are then examined and conclusions are drawn as to the usefulness of using Dr values to indicate changes along the case streets. Finally, comments are made concerning the links between the Dr values and the presence of physical features in the urban skyline.

Five visible skylines for each of 26 streets were traced from photographs selected to represent approximately 30 m intervals along each street. Figure 2 shows one of the photographs of Canning Crescent. A series of views similar to this was recorded along each case street and from these the skylines were traced. Figure 3 illustrates the skyline trace extracted from the photograph shown in Figure 2. It is made up of obliquely viewed buildings, regular chimney stacks, a small amount of vegetation and a telephone cable support.

Figure 2. Canning Crescent: a public housing development with a high degree of building uniformity, constructed *c.* 1930.

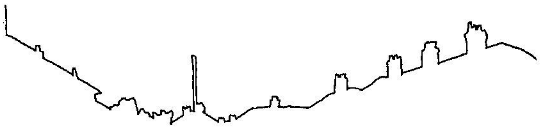

Figure 3. Canning Crescent skyline.

The traces were scanned and converted to a negative black and white image prior to measurement using proprietary software, Benoit 1.0 (Trusoft, 1997). Questions about suitable measurement parameters arose and four issues were indentified:

1. It has been suggested that fractal dimensions are calculated in relation to the upper

and lower size limits (*l*) of the subject. This is automatically done in Benoit 1.0 where the largest 'ruler' size is *l x* 0.25 and the subsequent 'ruler' sizes are reduced by a coefficient of 1.3.

2. However, the size of the subject skylines ranged in length from 276 to 1594 pixels and in height from 111 to 840 pixels. If the defaults of Benoit were used, the various skylines would be measured over differing ranges, making comparison difficult. There was also a concern that the lower level measurements of the smaller skylines would descend to pixel sizes below the widths of the traced lines and produce a distorted result.
3. There was a further concern that a standard range of measurements applied to all skylines would distort the results, pulling some Dr values up and others down.
4. If default values were to be related to *l*, should *l* be the height or the length of the trace?

To help resolve these issues it was decided to extract several sets of values across a number of ranges and examine them in relation to a set of measurable non-fractal characteristics present in the subject skylines. The results would be examined by computing correlations between the fractal dimensions found in each case and the values of the non-fractal characteristics. Table 1 lists the parameters used for the calculation of five sets of Dr values.

Urban skylines represent the combination of built and natural features, and five characteristics were identified that were intuitively felt to have potential connections with the skyline fractal dimension:

1. the degree of vegetation represented on the trace;
2. the degree of flat roof-line represented on the trace;
3. trace height;
4. trace length; and
5. trace height to length ratio.

Each of these characteristics was measured for each skyline and compared to the results of the Dr measurements. No individual correlation was found over $r = 0.709$ (set

Table 1. Dr value calculation parameters

Set 1	Benoit 1.0 defaults used. The largest ruler size was *l* x 0.25, where *l* was the height of each individual trace. Eight ruler sizes were used with a reduction coefficient of 1.3. The largest ruler size was 116 pixels and the smallest was 3 pixels.
Set 2	A standard range from 98 pixels down to 12 pixels was used with a reduction coefficient of 1.3 over 7 ruler sizes.
Set 3	The largest ruler size was set at *l* x 0.25, where *l* was the height of the trace. The lower limit was fixed at 10 pixels. The traces were measured with a reduction coefficient of 1.3. The largest number of rulers was 10 and the smallest 4.
Set 4	The largest ruler size was set using *l* x 0.25, where *l* was the length of the trace. Upper ruler sizes varied from 398 pixels to 67 pixels. The lower limit was set at 5 pixels (the actual width of the traced lines). Reduction coefficients were set at 1.3.
Set 5	A standard range of ruler sizes was set at 95 to 5 pixels. The upper limit represented a modal value in relation to the height of most traced lines. Reduction factors were again set at 1.3. The lower limit was set to avoid distortion caused by changes in the trace line thickness that might have been apparent below 5 pixels.

3 correlation with 'percentage of flat roofline'). However, set 5 produced the highest correlation over the range of characteristics, and the second highest scoring set was set 2 (both sets had standard ranges of ruler size). It was decided to use the best-fit set, set 5, for further examination.

Skyline Dr and physical features

The initial explanations of Dr were tested using correlation and regression analysis linking the Dr values to the presence of physical features in the urban skyline.

The individual skyline Dr values ranged from 1.3377 (Figure 4) to 1.0245 (Figure 5), with an average Dr of 1.1547, and a variance of 0.0036. As a check on accuracy, the standard deviation of residuals (Sdr) was calculated for each Richardson plot. The highest Sdr was 0.0830, which is low enough to warrant the use of the total Dr value as the primary indicator of fractal dimension.

The mean Dr calculated for each street, with five skylines per street, ranged from a maximum of 1.2580 to the lowest at 1.0902. In trying to determine reasons for the relative rank positions of the skyline sets by Dr, a visual examination of the skyline traces suggests that two factors are responsible: the presence of vegetation and the viewing angle. The four lowest-rank sets have the smallest amounts of vegetation present in their traces and all contain a large amount of flat roofline viewed straight on. In comparison, the eight highest-ranking sets all contain large amounts of mature vegetation or oblique views of the adjacent buildings or both. The traces of the skylines for the top three sets contain only mature vegetation. The traces that contained coniferous vegetation (Figures 6 and 7), although scoring higher than the traces with less vegetation, scored lower than those traces with a similar amount of deciduous vegetation. Though the trace in Figure 7 reflects the presence of coniferous vegetation, it is also affected by the presence of the outlines of deciduous trees which lifts the Dr value relative to that of the trace in Figure 6.

These observations are reinforced by Pearson correlation testing of mean skyline

Figure 4. Park Town. A very narrow visible skyline, dominated by mature vegetation, resulting in the highest Dr value (1.3377) for the whole set of skylines.

Figure 5. Warberg Crescent. In contrast to Figure 4 this shows a much wider visible skyline, dominated by flat-roofed buildings viewed straight on (Dr = 1.0245).

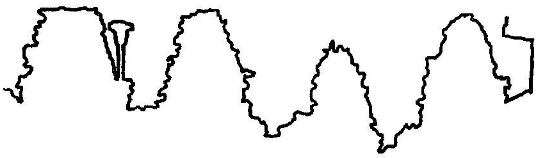

Figure 6. Park Close (Dr = 1.1321): a 1960s modernist development with a skyline dominated by coniferous vegetation.

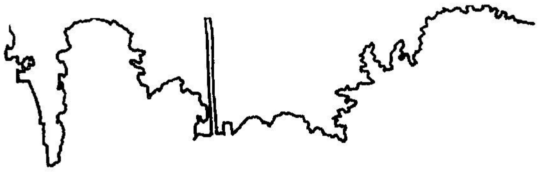

Figure 7. A skyline from Warnborough Road: a late-Victorian street of large houses set in mature gardens (Dr = 1.1573).

Dr (95 – 5 pixels) against amount of vegetation and flat roof-line. The tests reveal that as the percentage of vegetation on the skyline increases the Dr value also increases (r = +0.650, significant at the 0.01 level), and as the percentage of flat roof-line increases the Dr value decreases (r = -0.598, significant at the 0.01 level). The Dr value seems to represent the grain or texture – represented by the amount of vegetation or roof-line – of the skylines in relation to their height and width. The smaller traces with larger amounts of vegetation (such as Figures 8 and 9) have the higher Dr values. The wider traces, with a greater amount of flat roof-line (such as Figures 10 and 11) have lower Dr values. The amount of flat roof-line is influenced by viewing angle and those traces that present an oblique view down the street have a smaller flat roof-line percentage, and therefore higher Dr value, than those traces that present a roof-line viewed straight on.

The largest variation in Dr within each set – represented by the coefficient of variation (V) – is found in Tucker Road, where Dr values vary from 1.2793 (Figure 12) to 1.0550 (Figure 13). The higher value traces are those that contain mature vegetation, and oblique views, whereas the lower ones contain built elements viewed straight on. Figure 12 is the most oblique view of the set and contains foreground vegetation, whereas Figure 13 contains virtually no vegetation and is a view straight on to a flat-roofed industrial unit at the end of the street. These two figures illustrate the variation along a single street.

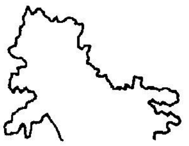

Figure 8. Bardwell Road (Dr = 1.2480): a street of mid-Victorian detached villas in mature gardens.

Figure 9. Park Town (Dr = 1.2724): views are severely restricted by mature trees.

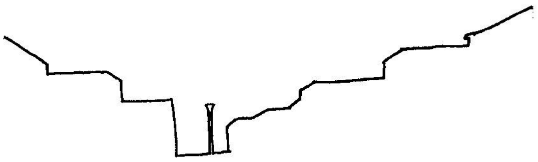

Figure 10. Park Close (Dr = 1.0478): a relatively wide skyline of a development dominated by flat-roofed buildings viewed obliquely.

Figure 11. Horseman Close (Dr = 1.0295): a 1960s development of terraced and semi-detached dwellings viewed straight on, producing a low level of irregularity and therefore low fractal dimension.

At the lower end of the range of variation, Mayfair Road presents a relatively uniform

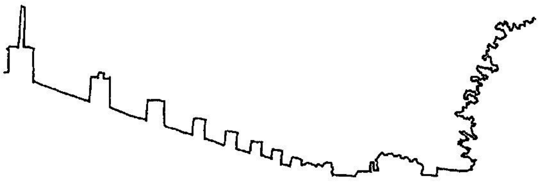

Figure 12. Tucker Road (Dr = 1.2793): narrow visible skyline, mixture of foreground vegetation and obliquely viewed roof-line producing a highly irregular trace.

Figure 13. Tucker Road (Dr = 1.0549): a wide visible skyline, an absence of vegetation, a small amount of oblique roof-line and an industrial warehouse building viewed straight on produces a relatively smooth trace.

Figure 14. View along Mayfair Road: a 1930s private sector speculative development of highly uniform semi-detached houses, set back from a very straight road line.

Figure 15. Mayfair Road (Dr = 1.1547): skyline produced from Figure 14.

set of skylines with a mean Dr of 1.1350. Figure 14 shows the view along Mayfair Road from which the skyline trace (Figure 15) is extracted. Figure 15 has a Dr of 1.1540, contains foreground vegetation and oblique views, and is the narrowest trace of the set. The skyline extracted from Figure 16 and presented as Figure 17 has a Dr of only 1.1027 and is again the trace with the lowest levels of vegetation and one of the least oblique views.

Figure 16. A second view along Mayfair Road showing slight difference in character resulting from the virtual absence of deciduous vegetation.

Figure 17. Mayfair Road (Dr = 1.1027): a wider visible skyline, a small amount of coniferous vegetation, an oblique roof-line, and roof-lines viewed straight on produces a lower fractal dimension than that shown in Figure 15.

It would appear that Dr values for skylines register the balance between built form and vegetation in relation to the dimensions of the visible skylines: generally, the higher the Dr value the greater the amount of vegetation in the view.

Conclusion

This paper has developed a method of calculating a Dr measurement for a street skyline, applied the technique to the assessment of a number of traced skylines, and sought to explain the resultant Dr values in relation to the physical characteristics of the streetscape. Dr is a composite measurement of the complexity and character of a street skyline, representing the combined

effect of several independent variables:
amount of vegetation;
amount of visible roof line;
orientation of buildings in relation to the view position;
width of view;
street height to width ratio; and
construction form of the visible roof-line.

For the skylines examined and the method employed, the fractal dimension ranges from 1.02 to 1.34. Dr is recording the mixture of built and natural forms in the urban skyline, ranging from skylines dominated by built form to those dominated by vegetation.

As calculated here, Dr is recording the degree to which the skyline fills the space it occupies, reflecting the amount of vegetation and flat roof-line and the size of the trace. Compared with the roof-lines, the outlines of deciduous vegetation, in leaf, are more irregular, smaller scale, and have a tighter texture – they fill a greater area over the same length. There is a tendency for a skyline with a high Dr to have a high degree of vegetation and an undulating roof-line, and be comprised of buildings that are viewed obliquely and occupy narrow plots. In contrast, there is a tendency for a skyline with a low Dr to have little or no visible vegetation, contain buildings with predominantly flat roofs with little or no undulation from either chimneys or gables, and present clear views straight on to buildings with wide frontages.

The images used as source data in this study were obtained during the summer season, when deciduous trees were in full leaf. The fractal dimension is likely to change seasonally, as the built form becomes more dominant after leaf fall in autumn, and less dominant in spring, after new leaf growth. The fractal nature of an urban skyline is dynamic, not just in terms of variation along a street, through changes in the ratios of physical elements, but also seasonally through changes in the natural environment.

Perhaps the most fundamental aspect of this fractal assessment, in relation to skylines, is that the Dr value is quantifying the degree to which the line representing a skyline fills the space it occupies: it is quantifying the 'wigglyness' of a visible horizon. Having discovered that this, quantifiable, degree of wigglyness relates to the mixture of built and natural forms, two possibilities emerge. First, fractal dimension can be used to compare the character of places in a quantifiable manner. Secondly, by understanding how the combination of physical features influences the degree of irregularity in a place, the recording of fractal dimension and its subsequent replication could be used to create new places with the same underlying level of irregularity as the old but without slavishly copying what already exists.

This exploratory paper presents an examination of fractal skylines as a starting point for further discussion of the potential of fractal analysis in urban morphology and urban design. Other possible applications of fractal analysis are to street networks, building elevations and street vistas. There is the potential application of fractal analysis to ground plans in relation to levels of attractiveness or even property values. Above all, it seems that fractal dimension allows quantification of qualitative attributes by providing a synthetic measurement of several variables that, in combination, affect the complexity and character of a place.

References

Alexander, C. (1977) *A pattern language* (Oxford University Press, New York).

Batty, M. (1995) 'New ways of looking at cities', *Nature* 377, 574.

Batty, M. and Longley, P. (1994a) 'Urban shapes as fractals', *Area* 19, 215-21.

Batty, M. and Longley, P. (1994b) *Fractal cities: a geometry of form and function* (Academic Press, London).

Batty, M. and Xie, Y. (1994) *Preliminary evidence for a theory of the fractal city* (NCGIA, State University of New York, New York).

Bechhoefer, W. and Bovill, C. (1995) *Fractal analysis of traditional housing: Amasya, Turkey* (University of Maryland, Baltimore).

Bovill, C. (1996) *Fractal geometry in architecture*