

An area structure approach to morphological representation and analysis

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Abstract. *A system for representing and analysing town plans as ‘area structures’ is presented. This starts from the observation that a town plan is a geometric object featuring areas whose relations can be articulated using diagrams and symbols. Devices such as the ‘containment graph’ and ‘area-stratum diagram’ are introduced, and a set of conventions for symbolic notation is demonstrated. These are used to represent morphological relations and operations. The paper suggests interpreting urban land, plots and building footprints as three ‘media’ (A, B, C) and uses symbolic notation to articulate typical urban morphological structures and relates these to existing urban form terminology. The area structure approach is used to interpret typical relationships at the block scale, including those between containment, subdivision, access and adjacency. Symbolic formulae are applied to characterize different kinds of urban morphology: onward linkage to ‘morphospace’ and design is suggested.*

Keywords: area structure, town-plan analysis, streets, plots, blocks, morphological structure, morphospace

Urban morphology, as a study of form in space, sooner or later involves the abstractions of mathematics. Some 30 years ago, Lord and Wilson suggested the need for a rigorous ‘*science of morphology*’ associated with a ‘*mathematics of form description*’ (1984, p. 7; original emphases). As we can consider ‘space as a mathematical concept, rather than as the site of our experience’ (Dahan-Dalmedico, 2011, p. 24), we can interpret the town plan – that classic device for urban morphological analysis – not only as geography, but *geometry*: a set of lines and areas on a plane inscribing an intricate structure (Figure 1).

That said, the town plan is not, as it were, a ‘flatland’ (Abbott, 2006) simply inhabited by

mutually exclusive polygons, with arbitrary patterns of adjacency and connection. There is a deeper morphological structure present, that we can infer from the geometry of the plan: a structure that is understood intuitively by urban morphologists, but not normally expressed explicitly.

This structure is manifested in the way that some areas are ‘contained’ within others; in the way that plots form plot series; that buildings *within* and *between* plots have different relations to each other; and that typically different patterns of access exist between adjacent buildings and plots. While these relationships are not inevitable, their typicality is pervasive enough to allow us to

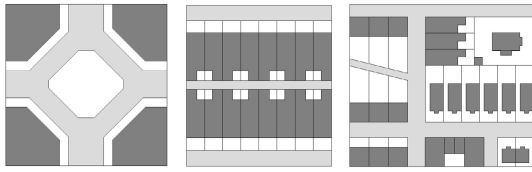


Figure 1. A town plan may be interpreted as a geometric object; but the morphological structure is more complex than a simple tessellation of polygons on a plane.

look at a town plan and infer a ‘generic structure’ (Kropf, 2014) or a kind of ‘urban syntax’, which marks out *urban* morphological structure from other kinds of structure (Marshall, 2009).

Consider the abstract geometry of Figure 2. The urban morphologist can readily imagine this as a portion of a town plan: where the three rectangles could be house footprints, and the three quadrilaterals garden enclosures. In fact, the urban significance here goes beyond individual areas: it concerns *sets* of areas: the three contiguous building footprints constitute a terrace, whose ground area intersects with the areas of the three different property parcels, such that each rectangle (house) is paired with a quadrilateral (garden), the whole set of areas forming a plot series. Together, this set of areas and their relationships can be considered an *area structure*. This structure, although abstract, is suggestive of the third dimension, distinctions of property and land use, probable access relationships, and possible ‘plannedness’. By investigating this structure more explicitly, it could help us better understand the nature of the urban fabric that is the object of urban morphological scrutiny.

Recently, Kropf has articulated ‘generic structure’ in terms of the basic urban morphological elements and their relationship in a ‘compositional hierarchy’ (2014). This makes relationships explicit in a way that provides a useful integration between the morphological traditions of Conzen, Caniggia and others. Meanwhile, it has been suggested that mathematics has the potential to transcend morphological traditions, and the use of symbols to express area structure relations

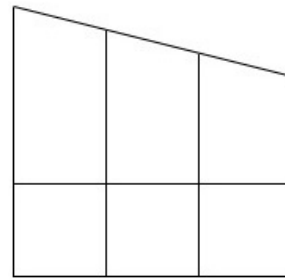


Figure 2. Geometric figure that could be an urban morphological structure.

between morphological elements has been demonstrated (Marshall, 2014). The present paper aims to develop the use of area structures to express urban morphological structure, and provide new explicit linkages between existing concepts and approaches.

In doing so, a response is offered to the wider morphographic agenda of Lord and Wilson (1984). A more general link is made to the use of diagrams and symbols to represent structures across the sciences, from molecular chemistry (Hoffmann and Laszlo, 1991) to the geometry of space-time (Barrow, 2008). A tradition is followed of interpreting morphology as a science of *possible* form, from the biological context of Goethe via D’Arcy Thompson (1917) to mathematical or quasi-mathematical interpretations of built form and their application to design (March, 1976; Steadman, 1983, 2014); a succession that invites application to the urban fabric.

The paper first recaps recent precedents in capturing urban morphological structure. Then area structure relations and morphological operations are demonstrated through diagrams and symbolic notation. The second half of the paper interprets these area structure representations with respect to urban morphology. This involves identifying three urban morphological ‘media’ – urban land, plots and building footprints – and their typical relationships. Relations to existing terminology are interpreted, and examples of typical area structures and their representation by diagrams and symbolic formulae are given. This system provides a platform for future morphological application.

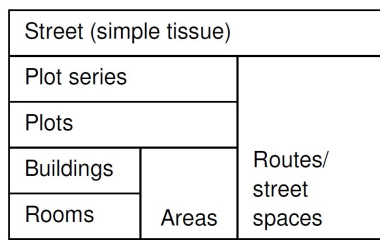


Figure 3. Excerpt from Kropf's (2014) multi-level diagram, expressing a hierarchy of morphological elements. This simplified version omits some smaller and larger scale elements.

Precedents: structure, syntax, graphs, codes and symbols

Urban morphological structure is sometimes interpreted as being complex, but in some ways its constitution is rather simple and familiar: relations between elements such as buildings, plots, blocks and streets are intuitively understood by any urban dweller, and have been familiar objects of study by urban morphologists for decades (Kropf, 2009; Moudon, 1997). If anything, the scholarly problem is that the general character of urban morphological structure is so familiar that its precise articulation has seemed unnecessary. And yet, there is a sense that urban morphological structure has not been completely or consistently grasped, amidst the different traditions and terminologies used to describe structure (Whitehand, 2012).

The idea that there are certain fundamental elements, such as streets, blocks, plots and buildings in relation to each other is a recurring theme in urban morphology (Caniggia and Maffei, 1979; Conzen, 1960; Habraken, 1998; Panerai *et al.*, 2004). From time to time there have been steps taken to integrate these. In a recent paper, significantly, Kropf (2014) provides both critical analysis and unpackaging of concepts, and a creative synthesis of distinct traditions. This aims to provide a 'common, central foundation for analysis, comparison and synthesis'.

Of particular note, Kropf expresses an urban

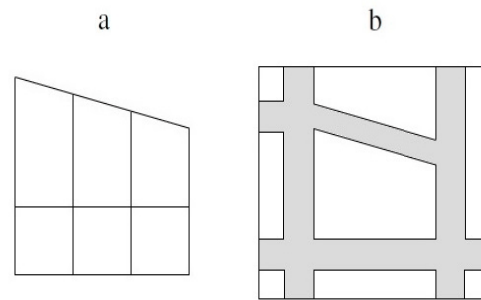


Figure 4. The distinction between (a) a 'jigsaw' and (b) an 'archipelago' (after Marshall, 2009, p. 73).

morphological hierarchy in the form of a multi-level diagram (2011, p. 395; 2014, p. 50) (Figure 3). This builds on Conzen's morphological hierarchy which Kropf identifies as fundamentally one of 'containment' (2014, p. 44): 'The building pattern is contained within the plot pattern, which is in turn contained within the street pattern. Looking closely at these two relationship shows, however, that they are not the same'. This invites exploration of the extent to which these are indeed similar or different.

On a related theme, the term 'urban syntax' has been used to refer to typical relationships between morphological elements: for example, buildings normally sit on plots, and plots normally access routes; hence typical rules of thumb may be identified (Marshall, 2009, pp. 73-7). We can also distinguish between 'jigsaws', referring to tessellations of mutually exclusive areas; and 'archipelagos' involving two different kinds of area, where one is carved from the other leaving a residual (Marshall, 2009, pp. 71-3) (Figure 4). While these relationships relate to the scale of streets and blocks, their pervasive nature underpinning the entire urban fabric enables them to be referred to as 'urban' syntax, just as the study of form at the level of streets and blocks is routinely referred to as 'urban' morphology, even if this is not the only scale at which morphological structure is identifiable. In fact, the geometric or topological relationships of interest here are not limited to any particular scale.

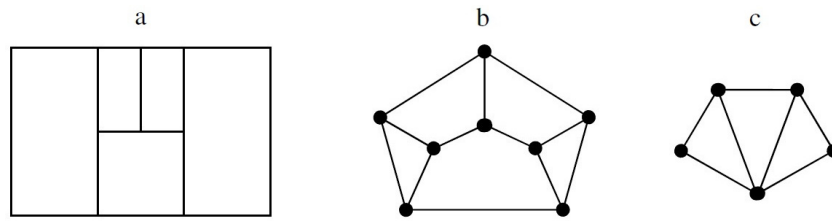


Figure 5. A building floor plan (a) and a topologically equivalent planar graph (b); after Lord and Wilson (1984, p. 183). In graph (c), the vertices correspond to the five constituent rectangles of (a), or polygons of (b), while the edges represent adjacency relations between those polygons.

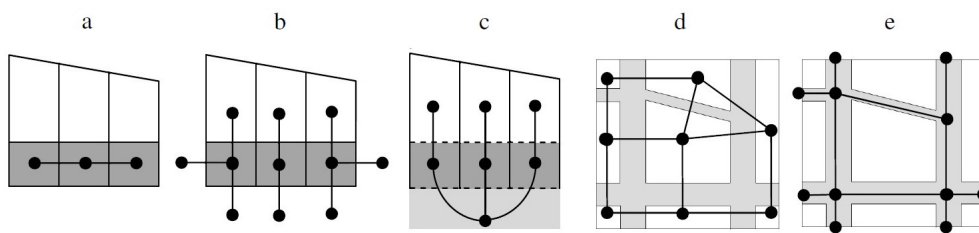


Figure 6. Five types of graph interpretation of the urban fabric: (a) party walls; (b) external faces; (c) access relationships; (d) blocks; (e) streets (after Krüger, 1979).

At the architectural scale, Steadman (1983) and Lord and Wilson (1984) have demonstrated the use of the *graph* as a means of capturing aspects of the structure of a building floor-plan (Figure 5). A graph is a kind of discrete mathematical entity familiarly represented as a diagram comprising vertices and edges (see, for example, Wilson, 2010) which has been applied in many disparate urban and spatial contexts (for example, Alexander, 1966; Barthélemy, 2011). Notably, Krüger (1979) has used graphs to articulate five types of spatial association between buildings, plots and streets (Figure 6). While graph-theoretical representations and analyses are commonplace in ‘spatial-analytical’ and ‘configurational’ morphological approaches, they are less routinely used in ‘historico-geographical’ or ‘typo-morphological’ traditions (as identified by Kropf, 2009). This invites consideration of further linkage.

Architectural morphologists have also devised ways of classifying and numerically encoding built forms (March, 1976; Steadman,

1998), which can potentially be extended to include the building situated on its plot and in relation to the street (Steadman and Marshall, 2005). Steadman has also demonstrated the enumeration of actual built forms in relation to a ‘morphospace’ of possible forms (1983; 2014). These approaches suggest the potential for extension to *urban* morphologies, that is, beyond the architectural scale.

Marshall (2014) has suggested recognizing the geometry of a town plan as an *area structure* – taken as a set of areas and their relationships. An area structure can be used as a model of selected properties of the urban fabric; as such it is analogous to ‘built form’ (see, for example, Krüger, 1979). Marshall (2014) has also suggested the use of symbolic notation to express relationships between areas. This symbolic notation can relate both to areas expressed in maps – and in principle any geometric representation – and also graph-theoretical relations. For example, ‘X|Y’ denotes ‘X abuts Y’, while ‘X|Y’ denotes ‘X accesses Y’; where X and Y are two areas (for example, plots or building footprints). Figure

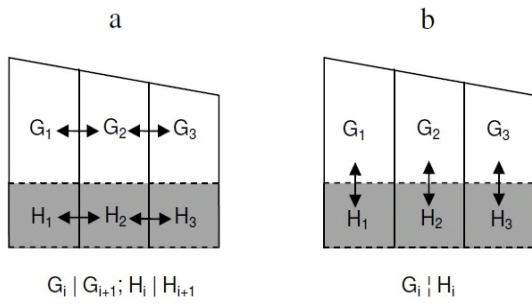


Figure 7. Symbolic notation applied to express (a) adjacency relationships; (b) access relationships, between houses (H_i) and gardens (G_i) (after Marshall, 2014).

7 shows examples of such relationships.

There is potential to seek integration between these morphographic approaches, and the relationships between the morphological elements and the relationships they express. This paper now develops these in more detail, to capture these relationships, expressed as area structures, articulated diagrammatically and expressed using a symbolic notation.

Area structure representations

The geometric object in Figure 5(a) is not just a set of five rectangles on a plane, but can be interpreted as an *area structure* comprising one large rectangle X containing three sub-rectangles X_1, X_2, X_3 , two further sub-rectangles, $X_{2,1}$ and $X_{2,2}$, and a further two sub-rectangles $X_{2,1,1}$ and $X_{2,1,2}$ (Figure 8). This interpretation implies successive relations of containment: we can use *containment structure* to refer to any set of elements and their relationships of containment.

We can represent the containment structure of Figure 8 by a graph, which we can refer to as a *containment graph* (Figure 9), since the relationships (represented by the arcs or edges) are relationships of containment – as opposed to relationships of access or adjacency, as in Figures 5(c) or 6.

Additionally, we can represent the structure by means of an *area-stratum diagram* (Figure 10). This new representation combines the structure of the containment graph (Figure 9) with the logic of Kropf’s multi-level diagram (Figure 3). This converts what Kropf created as an expression of generic hierarchy – for urban fabrics generally – into a specific representation of a particular configuration on the ground. Note that a horizontal baseline is indicated: this signifies where the area structure ‘meets the ground’ – the elements on the base layer ($X_1, X_{2,1,1}$, etc.) are what appear as mutually exclusive undivided polygons on the ground plane.

In fact, the letters with their subscripts themselves convey the sense of structure ($X_{2,1}$ is part of X_2 , etc.). We can also employ symbols to indicate containment relations between areas, such that an area X comprising sub-areas X_1 and X_2 is expressed using curved brackets: $X(X_1, X_2)$. In Figure 8, the full structure of containment relations can be expressed as: $X(X_1, X_2(X_{2,1}(X_{2,1,1}, X_{2,1,2}), X_{2,2}), X_3)$. This ‘containment formula’ – which may be likened to a chemical formula – is an expression of relationships that represents the whole ‘containment structure’ in a shorthand way.

Note that neither the formula, the containment graph nor the area-stratum diagram express a unique geometric or topological

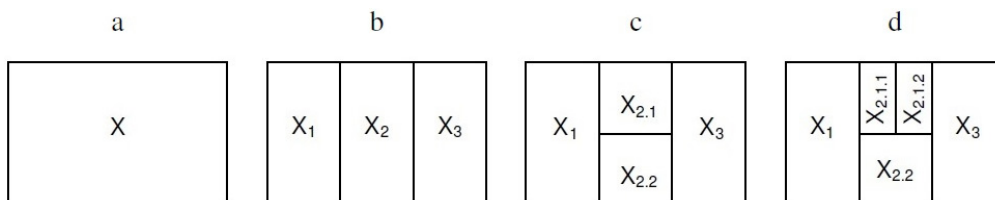


Figure 8. ‘Unpacked’ area structure of Figure 5(a). The labels with their subscripts are themselves indicative of structure.

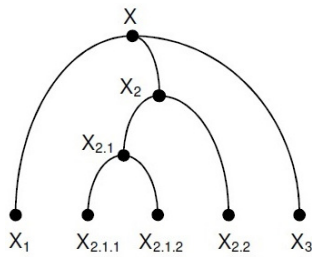


Figure 9. Containment graph for the structure in Figure 8.

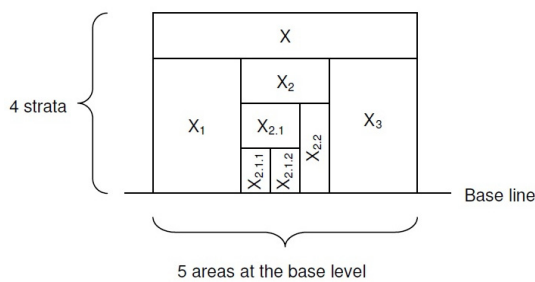


Figure 10. Area-stratum diagram, for the area structure in Figures 8 and 9.

configuration – we cannot tell if X_1 abuts X_3 or not (cf. condensed chemical formulae) – but they do tell us the hierarchical relations between areas, something which cannot definitely be inferred from the original geometry (Figure 5(a)). Rather, the original plan should be read in conjunction with the symbolic notation, containment graph or area-stratum diagram (Figures 8, 9 and 10) – which are three ways of saying the same thing – to gain the most complete understanding of structure.

In fact, the area structure represented in Figures 8, 9 and 10 is only one possible interpretation of the geometric figure given in Figure 5(a). Many alternative area structures would be possible – a selection is given in Figure 11 – each of which could be a valid representation of the reality on the ground. The choice of area structure – as with any model – is a matter of judgement, depending on which features and sub-structures are salient. In the urban context, salience could relate to urban function – or possibly order of development.

Area structure operations

Looking back, we could interpret a temporal order in the subdivisions in Figure 8 (reading from left to right) or Figures 9 and 10 (reading from top to bottom). In other words, the area structure represented by $X(X_1, X_2(X_{2.1}(X_{2.1.1}, X_{2.1.2}), X_{2.2}), X_3)$ could embed a succession of subdivisions: hence bringing the dimension of time into play. Indeed, the area structure approach can be used to express not only *relations* between areas, but *operations* over time. Change over time can be represented by an operation ‘becomes’, denoted by the arrow symbol ‘ \rightarrow ’. Here, we consider a series of operations, starting with subdivision and subtraction.

Subdivision or ‘slicing’ is when an area is sliced up into subdivisions that are intrinsically of the same kind, and that fill the entire original area. This gives rise to the ‘jigsaw’ formation (Figure 4(a)). For example, an element X could be divided into three sub-areas (Figure 12(a)). One could say $X \rightarrow [X_1], [X_2], [X_3]$; or more generally, $X \rightarrow [X_i]_n$, using square brackets to indicate areas that are subdivisions, where the subscript n indicates the number of subdivisions.

The term subtraction or ‘carving’ is here used to describe the action of partitioning in a way that separates the original figure into sub-entities of different types (Figure 12(b)), one being of the original kind. This gives rise to the ‘archipelago’ formation (Figure 4(b)).

For ‘subtraction’, we introduce a new expression X° , termed the residual, which is the same type of entity as X , while Y is a ‘new’ kind of entity. Here, $X^\circ = X - \{Y\}$, or more generally, $X^\circ = X - \Sigma\{Y_i\}$. For Figure 12(b), we can express this ‘carving’ operation as $X \rightarrow X^\circ \{Y_1\} \{Y_2\}$, or $X \rightarrow X^\circ 2 \{Y_i\}$. Here, the $^\circ$ symbol signifies something hollow from which something else has been removed. X° implies the existence of a higher order area X , just as any entity labelled X_1 implies the existence of a higher order entity X . The curly brackets each enclose a contiguous area of a type different from the original (bounding) area from which it is carved. Overall, this

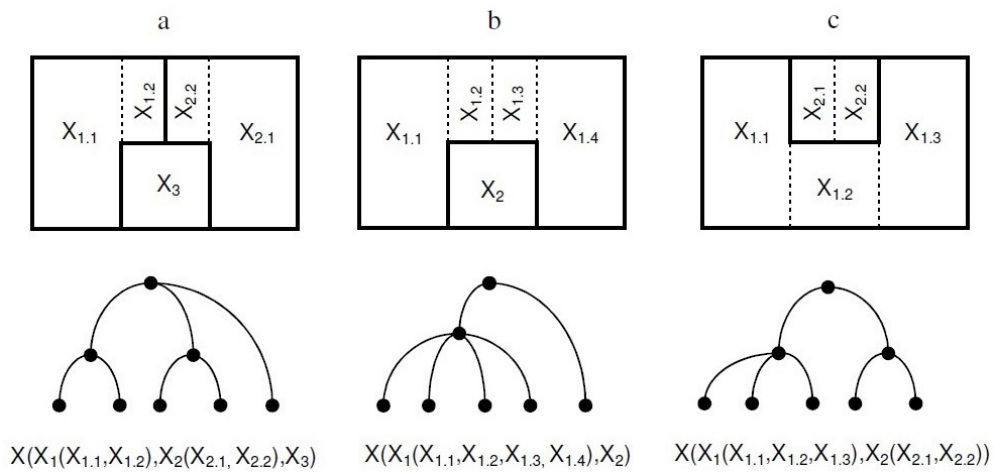


Figure 11. Alternative area structure interpretations of the same geometric figure.

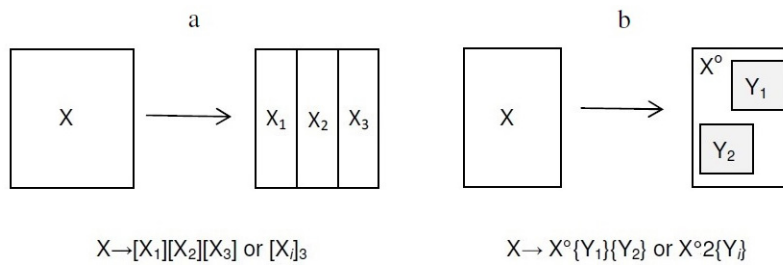


Figure 12. Basic area structure operations: (a) subdivision ('slicing'); (b) subtraction ('carving').

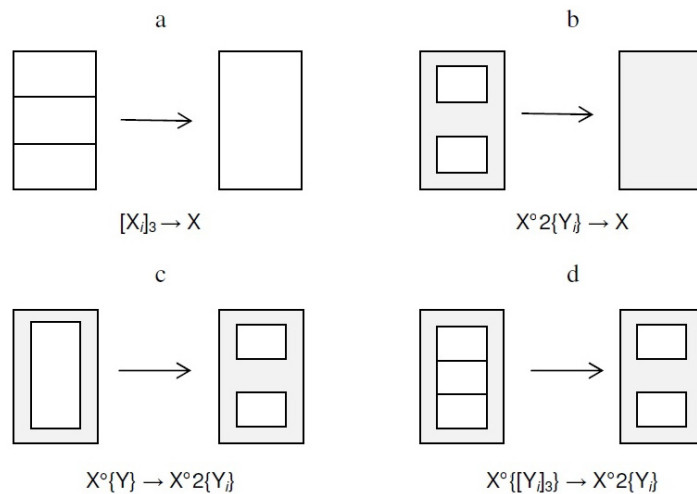


Figure 13. Further area structure operations: (a) amalgamation; (b) inundation; (c) incursion; (d) immersion.

notation captures the asymmetry implied between X° and Y with respect to X .

We can express further area structure operations, as shown in Figure 13. Here the English-language terms are not as important as differentiating the intrinsic kinds of area structure rearrangement involved.

Note that any area structure can be labelled to indicate its combination of slicing and carving, where:

- $[X_i]_m$ implies a single contiguous area comprising m subdivisions X_i ;
- $n\{X_i\}$ implies a set of n separately carved areas $\{X_i\}$.

Hence, in Figure 13(d) the left-hand configuration is given by $X^\circ\{[Y_i]_3\}$. This formulation features the four ‘base layer’ (ground plan) areas only, and is of itself a condensed version of a fuller containment formula $X(X^\circ\{Y([Y_i]_3)\})$ which additionally incorporates the information $X=X^\circ\{Y\}$ and $Y=[Y_i]_3$.

Urban media

Up till now, all the morphological structures considered herein could represent any kind of area structure in any context – from field boundaries to arrays of cells in a spreadsheet. Now, we move on to discuss more specifically the application to *urban* morphological structure.

As Kropf notes, ‘the plot would not seem to be an aggregate of buildings’ (2014). In fact we can suggest that plots and buildings are fundamentally *different* kinds of things. Similarly, plots and streets are fundamentally different kinds of things. We can recognize this distinction by considering that these are separate *media* which can take on different geometries. Hence each medium may be subject to subdivision (‘sliced’ to form a ‘jigsaw’) or subtraction (‘carved’ between two media into an ‘archipelago’). Having said this, all can be expressed as areas on plan, which have so far been represented by variations on the symbols X and Y .

Now, we can more specifically identify three urban morphological ‘media’ with

different but adjacent letters in the alphabet: A , B , C . Here, the general land area is labelled A (with streetspace the residual area A°); blocks and plots are labelled B (with residual unbuilt areas of a plot, such as yards or gardens, labelled B°); and building footprint areas are labelled C . This system is demonstrated in Figure 14 and explained in more detail below.

I. Let A represent the overall area of urban land under scrutiny. Let $B=\Sigma\{B_i\}$ represent the set of urban blocks, where individual blocks are labelled B_i . Let A° represent the residual space, which would include public streets, alleys and other public space: $A^\circ=A-\Sigma\{B_i\}$. Note that the term ‘residual’, while referring to what is a topologically ‘left over’ area, does not necessarily imply that it is functionally left over area, but could represent an element that has been deliberately constructed (as indeed the etymology of the word ‘street’ suggests).

II. Individual portions of streetspace can also be recognized, as subdivisions of A° , and labelled A°_i .

III, IV. Blocks may be divided into plot series and plots. For example, a block B_4 can be divided into plot series $B_{4.1}$, $B_{4.2}$ and $B_{4.3}$. Blocks or plot series may be divided into individual plots; hence $B_{4.1.1}$, etc. If a plot series constitutes a complete block B_p , then the plot series can be called $B_{i.0}$. If a plot constitutes a complete plot series $B_{i,j}$, then the plot can be called $B_{i,j.0}$. If a plot is both a complete plot series and a complete block, then it can be denoted $B_{i.0.0}$. In general, $X_{i.0.0}=X_{i.0}=X_i$; or $X_i(X_{i.0}(X_{i.0.0}))$.

V. A building footprint area C may be ‘carved from’ a plot or other area. This may lead to one or more residual spaces. A residual plot space can be given by $B^\circ=B-\{C\}$. B° represents the unbuilt area of a plot, such as a garden or yard: if there is more than one residual space, such as front and back gardens, these may be labelled B°_1 , B°_2 , etc. If the building fully occupies the plot, then there is no B° present. In the case of ‘encroachment’, a building footprint $\{C\}$ may also be carved directly from the streetspace: $A^\circ=A-\{C\}$. More than one building block may be placed on a plot: these may be labelled according to

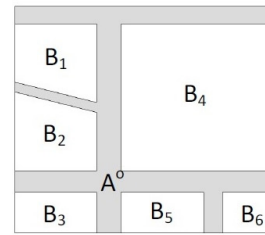
I. Subtraction of blocks from overall area

$$A \rightarrow A^\circ - \Sigma\{B_{ij}\}; \text{ or } A^\circ = A - \Sigma\{B_{ij}\}$$

A = overall ground plan area

A[°] = residual streetspace

Σ{B_{ij}} = set of blocks



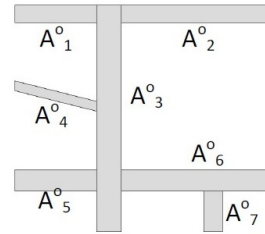
$$A \rightarrow A^\circ - \Sigma\{B_{ij}\}$$

II. Subdivision of streetspace into street segments

$$A^\circ \rightarrow [A^\circ_i]_n = A^\circ_{i1} + A^\circ_{i2} + \dots + A^\circ_{in}$$

A[°] = overall streetspace

A[°]_i = individual street segments



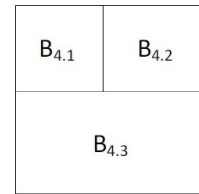
$$A^\circ \rightarrow [A^\circ_i]_7$$

III. Subdivision of blocks into plot series

$$B_{ij} \rightarrow [B_{ij,k}]_n = B_{ij,k1} + B_{ij,k2} + \dots + B_{ij,kn}$$

B_{ij} = block

[B_{ij,k}]_n = set of n individual plot series



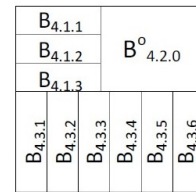
$$B_4 \rightarrow [B_{4,i}]_3$$

IV. Subdivision of plot series into plots

$$B_{ij,k} \rightarrow [B_{ij,k,l}]_n = B_{ij,k,l1} + B_{ij,k,l2} + B_{ij,k,l3} + \dots$$

B_{ij,k} = plot series

[B_{ij,k,l}]_n = set of n individual plots



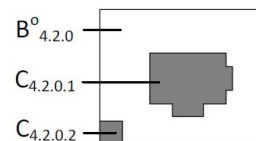
$$B_{4,1} \rightarrow [B_{4,1,i}]_3; \text{ etc.}$$

V. Subtraction of building block footprints from plots

$$B_{ij,k} \rightarrow B^\circ_{ij,k} - \Sigma\{C_{ij,k,l}\}$$

B[°]_{ij,k} = residual plot area (e.g. garden)

Σ{C_{ij,k,l}} = set of building block footprints

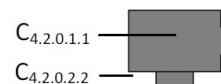


$$B_{4,2} \rightarrow B^\circ_{4,2,0} - \Sigma\{C_{4,2,0,i}\}$$

VI. Subdivision of building blocks

$$C_{ij,k,l} \rightarrow [C_{ij,k,l,m}]_n = C_{ij,k,l,m1} + C_{ij,k,l,m2} + \dots$$

C_{ij,k,l,m} = individual building footprints



$$C_{4,2,0,1} \rightarrow [C_{4,2,0,1,i}]_2$$

Figure 14. Area structure relations between and within different media (A, B, C).

Table 1. Examples of area structure interpretations of the *Glossary of Urban Form* (Larkham, (2014))

Urban form term	Area structure interpretation
<i>Basic elements and element complexes</i>	
Urban tissue (overall area)	$A = \Sigma C + \Sigma B^\circ + \Sigma A^\circ$
Element complexes	$\Sigma A^\circ, \Sigma B, \Sigma C$
Street system (element complex)	ΣA°
Street block	$\{B\}$
Plot pattern (element complex)	ΣB_i
Plot series (contiguous row of plots)	$[B_{i,n}]; n > 1$ (where $B_i B_{i+1}$)
Plot series (including adjoining streetspace)	$A^\circ \{[B_{i,n}]\}; n > 1$ (implicitly $A^\circ B_i$)
Building pattern (element complex)	ΣC_i
Block plan of a building (i.e. individual footprint)	C_i
<i>Specific elements or sub-elements</i>	
Street line	Line: $A^\circ \{\Sigma B\}$
Building line	Line: $A^\circ \{\Sigma C\}; \Sigma \{B^\circ C\}$
(Building line assuming frontage access)	Line: $A^\circ \{\Sigma C\}; \Sigma \{B^\circ C\}$
Plot boundary	Line: $B_i B_{i+1}$
Parent plot	$B \left\{ \right.$
Derivative plot	$[B_i]$ in context of $B \rightarrow [B_1, B_2, \dots]$
Medial plot	B_i in context of e.g. $A^\circ B(B_1, \dots, B_i, \dots, B_n)$; $1 < i < n$
Tail-end plot	B_n in context of e.g. $A^\circ B(B_1, \dots, B_n)$
Plot dominant	$C_1 \left\{ \right.$ in context of $B^\circ \{C_1, C_2, \dots\}$ or
Plot accessory	$C_i \left. \right\} B^\circ \{C_1\} \{C_2\}, \dots$; where $i > 1$
<i>Operations</i>	
Plot amalgamation	$\Sigma B_i \rightarrow B; \Sigma B_{i,j} \rightarrow B_i$; etc.
Subdivision	$B \rightarrow [B_{i,n}]; B_i \rightarrow [B_{i,j}]_n$; etc.; $n > 1$
Encroachment	$A^\circ \rightarrow A^\circ \{C\}$
Break-through street (i.e. ‘inundation’ or ‘incursion’, Figure 13).	$\left\{ \begin{array}{l} A^\circ \{B\} \rightarrow A^\circ \{B_1\} \{B_2\} \\ A^\circ \{B_1, B_2, B_3\} \rightarrow A^\circ \{B_1\} \{B_3\} \\ A^\circ \{C_1, C_2, C_3\} \rightarrow A^\circ \{C_1\} \{C_3\} \end{array} \right.$

the plot they occupy, for example building blocks $C_{4.2.0.1}$ and $C_{4.2.0.2}$ on plot $B_{4.2.0}$.

VI. A building block footprint C_i may be subdivided into individual building footprints $C_{i,j}$.

We can relate these area structure interpretations to existing morphological terminology. For example, in Conzenian terms, ΣA° in effect corresponds with the street system; ΣB with the plot pattern, and ΣC represents the building pattern; the labelling convention introduced here (A, A° , B, B° , C) reflects the

essential topological relationships between areas. To take a more detailed instance, Conzen asserts that ‘A row of plots, placed contiguously along the same street-line, each with its own frontage, forms a *plot series*’ (1960, p. 5). Here, we can interpret a row of contiguous plots as $[B_{i,n}]$; and their placement along a common street-line as $A^\circ \{[B_{i,n}]\}$ (implicitly with frontage access; $A^\circ | B_i$). Table 1 shows suggested area structure interpretations relative to a wider set of terms in the *Glossary of Urban Form* (Larkham, 2014).

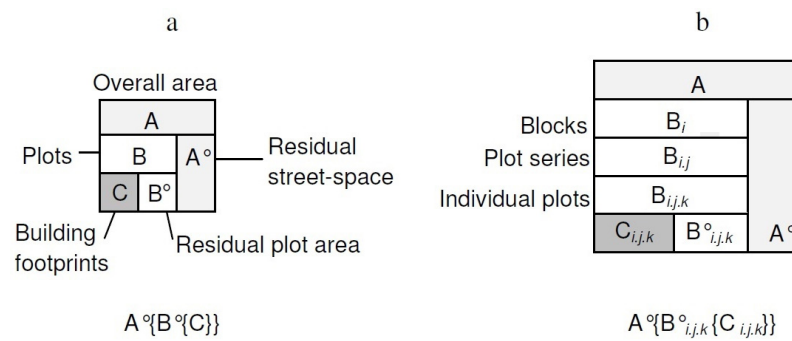


Figure 15. Kropf-style multi-level diagram expressed in three-media (A, B, C) format for two levels of resolution (a), (b). The items that appear explicitly in the formula are those on the base level of the diagram.

While the correspondence between terms in the table may not be definitive – contingent on how original terms are interpreted – the symbolic notation at least serves to invite renewed scrutiny of those definitions and their interpretations.

Urban syntax

This approach can help to articulate ‘urban syntax’ (Marshall, 2009) in terms of the distinctive and typical relationships between A, B and C that give urban morphological structure its distinct and typical urban character – as opposed to any generic geometric structure. In a sense the ABC are ‘media’ that intersect with the abstraction of area structure and give it urban meaning. While this analysis is focused at the block level, this ‘urban’ (as distinct from ‘architectural’) syntax could in principle characterize an entire urban area, just as a chemical formula can in principle denote the composition of a substance of any physical extent.

We could generalize by saying that urban morphological structure (as opposed to any mathematical area structure) has a typical form given by the expression $A^\circ\{B^\circ\{C\}\}$. By saying ‘typical’, this does not mean that alternative structures do not exist, but that these would be unlikely to occur as urban structures in practice, and only then as special

cases. For example, it is theoretically possible to have a structure $B^\circ\{A\}$ but this would imply a section of street isolated within a plot; the structure $C^\circ\{C\}$ would imply a building inside a building – theoretically possible, perhaps, but not a typical urban morphological structure.

While the generic containment structure $A^\circ\{B^\circ\{C\}\}$ may be intuitively understood by urban morphologists, it is not a relationship that is often remarked upon – for example, that the equivalence of the expressions $A^\circ\{B\}$ and $B^\circ\{C\}$ (that is, both of the form $X^\circ\{Y\}$) makes it clear that the relationship between A° and B is the same as that between B° and C (but different from that between blocks and their subdivisions, B_i and $B_{i,j}$).

Here, the use of a symbolic notation strips away the context of urban form and function: what is public and private, what is a street (associated with public passage), what is a plot (often a garden or landscaped area or perhaps car park) and what is a three-dimensional building (for example, the difference between lines representing walls and those representing kerbs); and focuses on the topological relationship. However, in return the abstract relationship sheds further light on the nature of those very real relationships.

The generic structure of this urban syntax can be demonstrated by applying it to Kropf’s multi-level diagram, converted to the three-media ABC format in Figure 15. Note that the

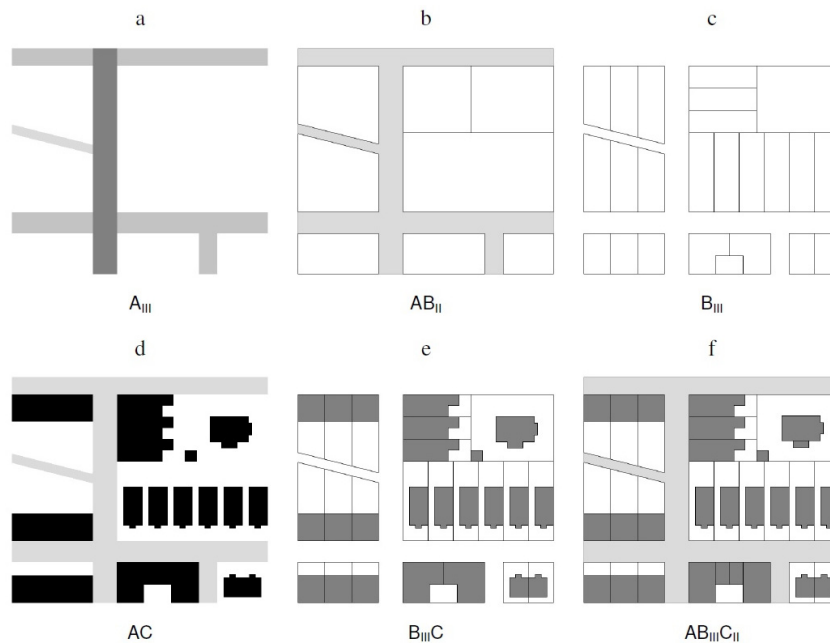


Figure 16. Different types of ‘morphology’ addressing the same urban fabric.

lowest level indicates what we would find explicitly visible on the ground plan – though in some cases, any of B_i , $B_{i,j}$ or $B_{i,j,k}$ could also be present on the ground plan.

Classification of types of morphology

We can use an ABC notation to classify different ‘morphologies’ in the sense of different types or levels of morphological focus. The immediately preceding discussion assumed recognition of three urban media (A, B, C), the middle of which is articulated over three levels (Figure 15 (b)). This can be classified as an $AB_{III}C$ morphology. Earlier, Figure 14 additionally distinguished between building blocks and individual building footprints: this can be classified as an $AB_{III}C_{II}$ morphology. A morphology omitting recognition of plot series could be labelled $AB_{10I}C$. A transport planning (or space syntax) approach might focus only on streets (type A) with no attention to the content of the blocks: differentiating three levels of road hierarchy would be A_{III} . An urban design figure-ground study that made no reference to streets, plots or

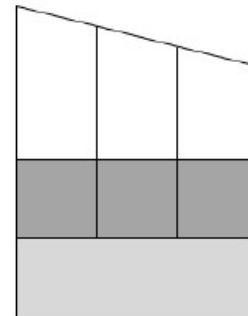


Figure 17. An area structure.

blocks would be type C. As such, the symbolic articulation clarifies explicitly different kinds of ‘morphology’ relative to each other (Figure 16).

Containment, access and adjacency relations

We can suggest how containment structure may relate to adjacency and access relations. Consider the area structure in Figure 17. This can be ‘unpacked’ into 14 salient elements,

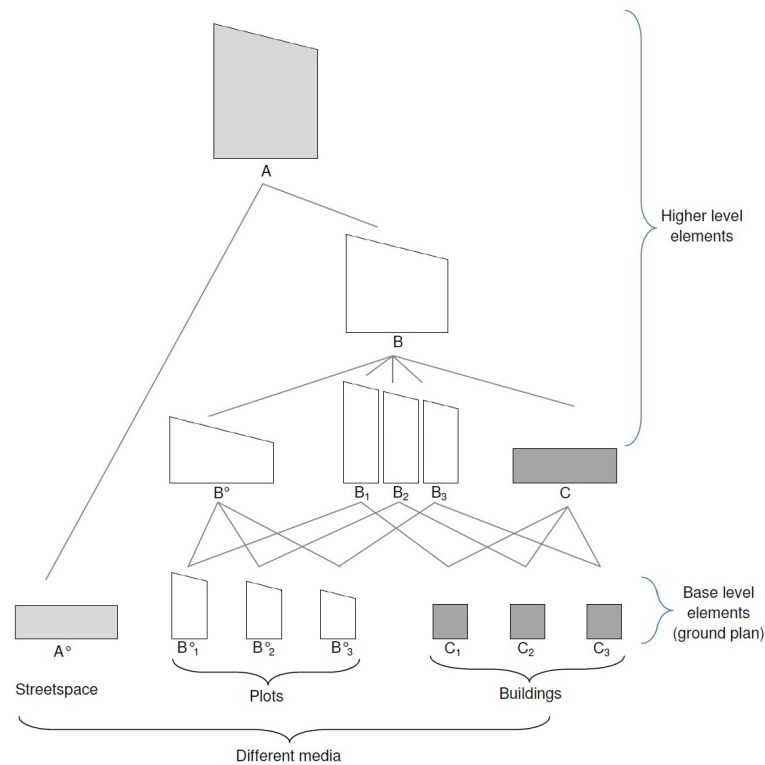


Figure 18. The area structure in Figure 17 ‘unpacked’ to reveal a containment structure of 14 salient elements and their relationships.

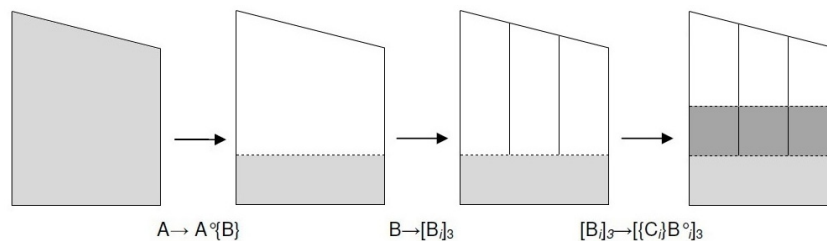


Figure 19. Carving followed by slicing followed by carving.

whose containment relationships are articulated in Figure 18.

Figure 18 demonstrates that the original area structure (Figure 17) is not just a ‘flat’ tessellation but embodies more than just what is manifest at the base layer (that is, A° , B°_1 – B°_3 , C°_1 – C°_3); that other ‘higher level’ elements are implied parts of the structure too (that is, including the set of gardens B° , individual properties B_1 – B_3 , and the terrace C).

Hence this shows how urban morphological structure is not just about the ground plan, but what ‘goes into’ the ground plan.

On further inspection, an interesting feature here is that there is a relationship between the slicing/carving structure (Figure 19) and the access/adjacency pattern (Figure 20):

(i) Either a slice or a carve results in new areas or sub-areas that are necessarily adjacent to and contiguous with each other (for example,

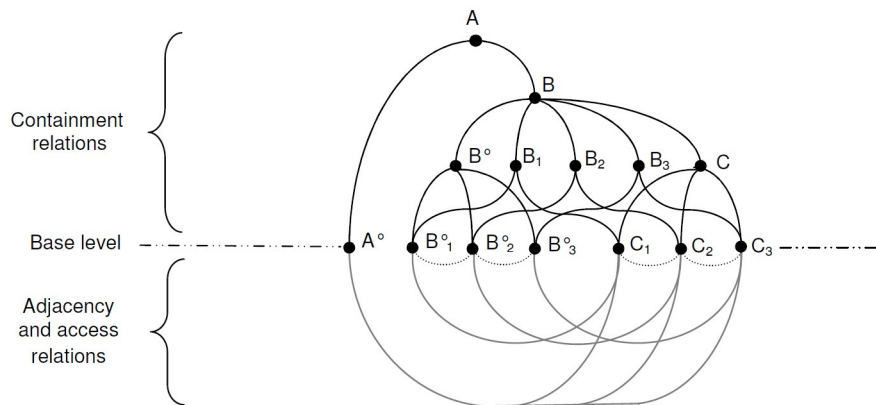


Figure 20. Containment graph (cf. Figure 18), appended with four adjacency relationships (dotted grey arcs, cf. Figure 7(a) and six access relationships (solid grey arcs, cf. Figure 6(c)).

$B \rightarrow [B_i]_3$; and $B_i | B_{i+1}$) – since they were originally contiguous;

(ii) Slicing *tends to* result in ‘boundaries’, that is, areas or sub-areas that do not have direct access between them – for example, walls between gardens or party walls between adjacent houses (that is, $B^\circ_i | B^\circ_{i+1}$ and $C_i | C_{i+1}$) (cf. Figure 7(a));

(iii) Carving *tends to* result in ‘borders’, that is, areas or sub-areas that have direct access between them – for example, the ability to move from the public street to a plot, or from a plot into the building on the plot (that is, $A^\circ | B^\circ_i | C_i$) (cf. Figure 7(b)).

It is suggested that the two ‘tending’ relationships (ii, iii) are typical for urban morphology – but this is not necessarily the case. That is, whereas slicing and carving *necessarily* result in areas that are adjacent, the access relations are dependent on context. These suggested ‘typical’ relationships can, of course, be subjected to empirical testing.

Characterization of urban morphological structure

Morphological formulae may be used to specify the content of an area structure at different degrees of resolution. Table 2 shows four formulae for expressing each area structure given in Figure 21. The first formula

gives a full set of containment relations; the rest are successively simplified or condensed. In doing so, formulae can take advantage of the regularity of the typical urban syntax. In particular, where blocks are entirely constituted by linear arrays of street-fronting plots, and where there is a linear progression of unbuilt (B°) and built (C) and areas from front to back of the plot, then we can specify block structures as linear strings. For example:

- A single block could have the formula $A^\circ \{n[B^\circ \{C\} B^\circ]_m\}$, where n is the number of plot series, and m is the number of plots per plot series;
- A series of blocks with a uniform plot series could have the formula $A^\circ n \{[B^\circ \{C\} B^\circ]_m\}$, where n is the number of blocks, and m is the number of plots per plot series. For example, in the case of Figure 21(b), $n=2$ and $m=8$.

More complex formulae would occur where there were different values of plots per plot series and plot series per block, and different numbers of buildings per plot, and so on.

Clearly, as morphological structure becomes more complex, the notation soon becomes unwieldy. In practice, rather than specifying areas across whole fabrics, this kind of notation is most likely to be used to check the spatial logic of parts of urban fabrics, or to extract types and make comparisons between different urban fabrics.

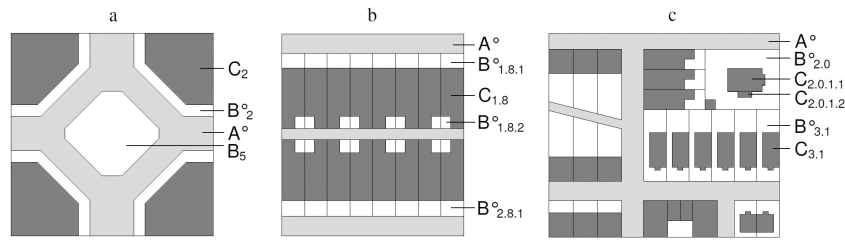


Figure 21. Urban block morphologies.

Table 2. Alternative formulae for area structures in Figure 21

	(a)	(b)	(c)
Full containment formula ¹	$A(A^{\circ}4\{B_i(B^{\circ}\{C_i\})\}\{B_5\})$	$A(A^{\circ}2\{B_i([B_{ij}([B^{\circ}_{ij,1}\{C_{ij}\} B^{\circ}_{ij,2}])])_8\})$	$A(A^{\circ}\{B(B_1([B_{1,i}([\{C_{1,i}\}B^{\circ}_{1,i}])])_3), B_2([B_{2,0}([B^{\circ}_{2,0}\{[\{C_{2,0,1,i}2\}\{C_{2,0,2}\}])])_2), B_3([B_{3,i}([B^{\circ}_{3,i}\{C_{3,i}\}])])_6\})\})$
Cadastral formula ²	$A^{\circ}4\{B^{\circ}_i\{C_i\}\}\{B_5\}$	$A^{\circ}2\{[B^{\circ}_{ij,1}\{C_{ij}\}B^{\circ}_{ij,2}]_8\}$	$A^{\circ}\{[\{C_{1,i}\}B^{\circ}_{1,i}]_3, [B^{\circ}_{2,0}\{[\{C_{2,0,1,i}2\}\{C_{2,0,2}\}]\}_2, [B^{\circ}_{3,i}\{C_{3,i}\}]_6\}$
Condensed formula ³	$A^{\circ}4\{B^{\circ}\{C\}\}\{B\}$	$A^{\circ}2\{[B^{\circ}\{C\}B^{\circ}]_8\}$	$A^{\circ}\{[\{C\}B^{\circ}]_3, [B^{\circ}\{C_2\}\{C\}], [B^{\circ}\{C\}]_6\}$
Elemental formula ⁴	AB_5C_4	$AB_{32}C_{16}$	$AB_{10}C_{12}$

Notes

- ¹ Indicates the containment structure, identifying explicitly each area and sub-area in the structure.
- ² Indicates the containment structure, identifying explicitly each area (recorded on the ground plan).
- ³ Indicates the number of areas in the containment structure (analogous to condensed structural formula in chemistry).
- ⁴ Indicates the total number of areas of each type (analogous to a molecular formula). Summing subscripts (where no subscript, this taken as 1) gives total number of areas at the base layer.

Figure 22 shows some examples of typical plot series: for example, case (a) is detached housing; (f) is semi-detached housing; the remainder are terraces; the two right-hand columns include outbuildings. Cases (n) and (o) – implying contiguous rows of out-buildings – may not be typical but are included for completeness. Figure 22 uses a further condensed formula (omitting brackets) which uses B_m or C_m to indicate contiguity (of m elements of the same type) across plot boundaries. For example, in case (a), $B^{\circ}_4.4C$ implies that the four areas B° are contiguous, whereas the four areas C are not: the reverse applies in case (b).

Urban block ‘morphospace’

Area structures can be used not only for interpreting existing urban morphological structures, but exploring the ‘morphospace’ of possible permutations of form (Steadman, 2014). Figure 23 could be interpreted as a morphospace of plot series; Figure 23 shows a morphospace of block configurations with a 3 by 3 array of tilings, having the generic structure $A^{\circ} w\{[B^{\circ} x\{[C]_y\}]_z\}$ – simplified to $w[B^{\circ} xC]_y$ in Figure 23 – where w , x , y and z are case-specific variables, relating to number of blocks (w), building footprints (x), buildings

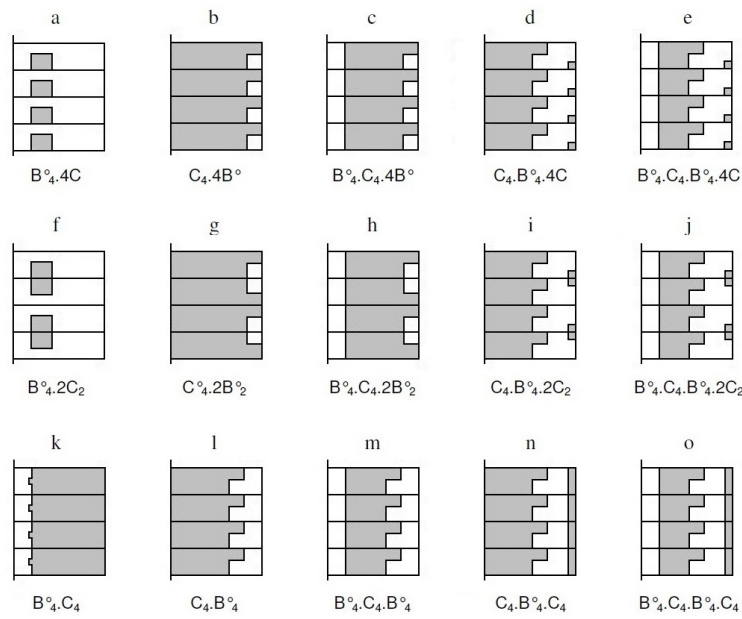


Figure 22. Plot series configurations (using condensed plot series notation).

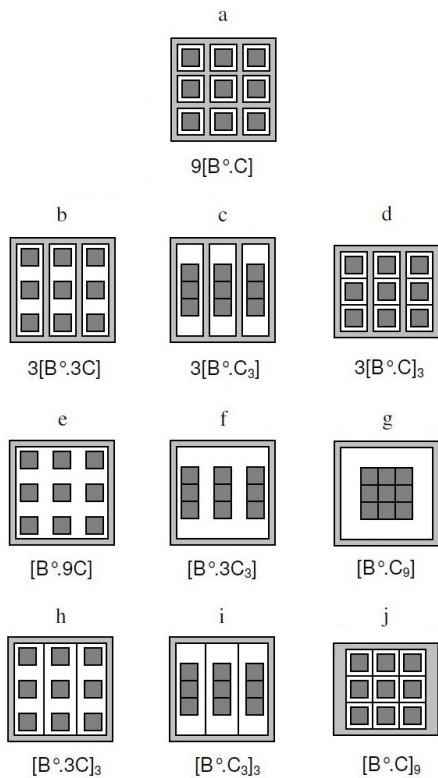


Figure 23. Urban block morphospace: ten possible permutations of the urban syntax $w[B^\circ xCy]z$ to give a square array of nine buildings (such that $w.x.y.z=9$).

per building footprint (y) and plots per plot series (z), which in this case (Figure 23) can each be 1, 3 or 9, but always such that the product $w.x.y.z = 9$.

These cases make manifest the idea of morphology as the study of possible forms (Steadman, 1983, 2014). This approach can be used both analytically to relate actual forms to each other (typologically or historically), and creatively to explore ‘design space’ – to generate *possible* forms for use in urban design and planning.

Conclusions

Urban morphological structure, as traditionally represented through the geographical device of the town plan, can in turn be interpreted in geometric terms as an area structure. The area structure takes us beyond the town plan as a two-dimensional plane, but can be unpacked as a ‘containment structure’ manifested through different urban ‘media’, articulating typical relations such as subdivision, access and adjacency. Diagrams and formulae have enabled articulation of the typical urban syntax – of streetspace containing plots containing

buildings. While this relationship is intuitively well known, the formula $A^\circ\{B^\circ\{C\}\}$ captures the sense of containment explicitly. The related expression $A^\circ\{B^\circ_{i,j,k}\{C_{i,j,k}\}\}$ recognizes the structural *similarity* between the successive subdivisions of B_i , $B_{i,j}$ and $B_{i,j,k}$ (respectively blocks, plot series and plots) while recognizing the structural *difference* between these and the relations between $A^\circ\{B\}$ and $B^\circ\{C\}$. The symbolic articulation can also embed a chronological order, allowing the morphographic description to encode morphogenetic knowledge.

This paper has demonstrated use of the *containment graph* (beyond the five types of graph of Krüger, 1979); the *area-stratum diagram* (building on the multi-level diagram of Kropf (2011, 2014), converted to apply to particular configurations); and corresponding *symbolic notations* (going beyond Marshall, 2014, by relating to containment relations and operations, and the three urban 'media' A, B, C; and expressing overall structure). This treatment simultaneously extends and cements the value of those existing diagrams and notations.

The notation has been related to existing urban morphological terminology (Larkham, 2014). This allows comparative interpretations: the type of area labelled B is the same whether called a plot or lot or *lotto*; and B° is the same whether a yard, garden or *area di pertinenza* (Kropf, 2014, p. 46). Hence, the symbolic area structure treatment can act as a bridge between languages and morphological traditions.

Overall, this paper helps to integrate area structure (Marshall, 2014) with 'urban syntax' (Marshall, 2009) and 'generic structure' (Kropf, 2014; incorporating previous interpretations of Conzen, Caniggia and others), while also linking with ideas for representing and coding built form (cf. March and Steadman, and others) and architectural morphospace (Steadman, 2014); so helping bridge not only between different urban morphological traditions but linking to architectural morphology and, potentially, design. While this paper has focused on the urban block scale, the area structure approach could be extended to

architectural morphology by including internal floorspace D (and its complement, the solid building fabric such as walls, C°) and the distinction between enclosed rooms (E) and circulation space (D°), hence implying $A^\circ\{B^\circ\{C^\circ\{D^\circ\{E\}\}\}\}$.

Future research is suggested on six fronts: (i) formal expression of the mathematical nature of area structure, relative to other kinds of geometry; (ii) a fuller exploration of urban 'morphospace'; (iii) interpretation of a fuller set of area structure relations and operations (for example, over wider urban scales, or three dimensional built form); (iv) development of area structure morphometrics (for example, quantification of aspects of containment structure); (v) onward application (or retrofit) to empirical study of urban fabrics, and their urban functions; and (vi) linkage to morphogenetic processes, including design and planning.

Overall, the area structure approach can bridge between the town plan as a site of geographical experience and a model of geometric possibility. This is at heart a morphographic project, which although abstract, links morphology, morphospace and morphogenetics – potentially bridging between science, imagination and future action.

References

- Abbott, E. A. (2006) *Flatland: a romance of many dimensions* (Oxford University Press, New York).
- Alexander, C. (1966) 'A city is not a tree', *Design* 206, 46-55.
- Barrow, J. D. (2008) *Cosmic imagery: key images in the history of science* (Bodley Head, London).
- Barthélemy, M. (2011) 'Spatial networks', *Physics Reports* 499, 1-101.
- Caniggia, G. and Maffei, G. L. (1979) *Composizione architettonica e tipologia edilizia 1: lettura dell'edilizia di base* (Marsilio, Venezia).
- Conzen, M. R. G. (1960) *Alnwick, Northumberland: a study in town-plan analysis*, Institute of British Geographers Publication 27 (George Philip, London).
- Dahan-Dalmedico, A. (2011) 'Mathematics and the sensible world', in Legendre, G. L. (ed.) *Mathematics of space* (Wiley, London) pp. 18-27.

-
- Habraken, N. J. (1998) *The structure of the ordinary: form and control in the built environment* (MIT Press, Cambridge, MA).
- Hoffmann, R. and Laszlo, P. (1991) 'Representation in chemistry', *Angewandte Chemie* (International Edition in English) 30, 1-16.
- Kropf, K. (2009) 'Aspects of urban form', *Urban Morphology* 13, 105-20.
- Kropf, K. (2011) 'Morphological investigations: cutting into the substance of urban form', *Built Environment* 37, 393-408.
- Kropf, K. (2014) 'Ambiguity in the definition of built form', *Urban Morphology* 18, 41-57.
- Krüger, M. J. T. (1979) 'An approach to built-form connectivity at an urban scale: system description and its representation', *Environment and Planning B: Planning and Design* 6, 67-88.
- Larkham, P. J. (2014) *Glossary of urban form* (www.urbanform.org/glossary.html).
- Lord, E. A. and Wilson, C. B. (1984) *The mathematical description of shape and form* (Ellis Horwood, Chichester).
- March, L. (1976) 'A Boolean description of a class of built forms', in March, L. (ed.) *The architecture of form* (Cambridge University Press, Cambridge) pp. 41-73.
- Marshall, S. (2009) *Cities, design and evolution* (Routledge, Abingdon).
- Marshall, S. (2014) 'A symbolic articulation of morphological structure', *Urban Morphology* 18, 73-5.
- Moudon, A. V. (1997) 'Urban morphology as an emerging interdisciplinary field', *Urban Morphology* 1, 3-10.
- Panerai, P., Castex, J., Depaule, J. C. and Samuels, I. (2004) *Urban forms: the death and life of the urban block* (Architectural Press, Oxford).
- Steadman, J. P. (1998) 'Sketch for an archetypal building', *Environment and Planning B: Planning and Design* 27, 92-105.
- Steadman, P. (1983) *Architectural morphology. An introduction to the geometry of building plans* (Pion, London).
- Steadman, P. (2014) *Building types and built forms* (Matador, Kibworth Beauchamp).
- Steadman, P. and Marshall, S. (2005) 'Archetypal layout: extending the concept of the archetypal building to streets and block layout', unpublished paper presented to the SOLUTIONS conference, London, December.
- Thompson, D. W. (1917) *On growth and form*. (Cambridge University Press, Cambridge).
- Whitehand, J. W. R. (2012) 'Issues in urban morphology', *Urban Morphology* 16, 55-65.
- Wilson, R. J. (2010) *Introduction to graph theory* 5th edn (Prentice Hall, Harlow).
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Inheriting the city: advancing understanding of urban heritage

This conference on urban heritage will be held in Taipei, Taiwan from 31 March to 4 April 2016. Urban heritage would appear to be an increasingly important asset for communities and governments alike, allowing cities to mark their distinctiveness and attract tourists and investment. At the same time the heritage of the future is being created. This conference seeks to examine the processes of protecting, planning and promoting urban heritage in the face of changes, pressures and opportunities globally and locally. The aim is to increase understanding of ways in which heritage can be used in the development of cities.

The conference will cross disciplinary boundaries and papers are invited from a variety of disciplines, including anthropology, architecture, archaeology, art history, cultural geography, cultural studies, design, ethnology and folklore, economics, history, heritage studies, landscape

studies, leisure studies, museum studies, philosophy, political science, sociology, tourism studies, urban history, and urban/spatial planning.

Themes of interest include:

- Innovative methods of protecting and planning urban heritage
- Community approaches to and uses of urban heritage
- Urban heritage as a form of social resistance
- Heritage as city memory
- Cosmopolitan urban heritage and the re-creation of identities
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Potential contributors should send a 300 word abstract, including a clear title and contact details, to ironbridge@contacts.bham.ac.uk by no later than 15 October 2015.
